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ANOMALOUS DIMENSIONS OF TRANSVERSE-MOMENTUM DEPENDENT PARTON DISTRIBUTION FUNCTIONS*

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We discuss recent developments in the understanding of gauge-invariant transverse-momentum dependent (TMD) parton-distribution functions (PDF). We compute the leading-order $\overline{\text{MS}}$ -scheme anomalous dimension of such a quantity in the light-cone gauge and show that it receives a contribution that can be associated with a cusp obstruction at transverse light-cone infinity. This anomalous dimension is intimately related to expectation values composed of fields and eikonal factors along a cusped contour and is absent in covariant gauges. The implications of these findings are addressed and a modified definition of TMD PDFs is proposed.

1 Introduction

In this report, recent theoretical results on TMD PDFs are presented, giving particular emphasis on their renormalization-group properties in conjunction with local color-gauge invariance. The latter is ensured by the presence of Wilson lines (gauge links) that are

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required to obtain gauge-invariant definitions of PDFs, integrated over the parton transverse momentum, and also unintegrated, i.e., TMD PDFs (see, for reviews [1, 2]). These quantities encapsulate the nonperturbative parts of the QCD processes, being themselves universal, with a (large) momentum-scale dependence controlled by perturbative QCD in the form of renormalization-group type evolution equations, like the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equation.

Central to the solution of these equations is the knowledge of the appropriate anomalous dimensions, calculable within perturbative QCD. One of the main purposes of this presentation is to discuss the physical impact of the leading-order anomalous dimensions in the light-cone gauge of matrix elements containing local operators and path-ordered exponentials along gauge contours that comprise transverse segments extending to light-cone infinity. The particular relevance of the anomalous dimensions originates from the fact that, in contrast to the gauge links which are path-dependent, they are local objects and ensure the gauge invariance of contour-dependent operators in terms of “logarithmic”, i.e., additive, Ward-Takahashi/Slavnov-Taylor identities. Because the anomalous dimensions of such operators ensue from obstructions, like endpoints, cusps, or self-intersections, one is able to analyze the renormalization-group properties of TMD PDFs solely by means of these quantities without any reference to the explicit, in general complicated, gauge-link structure. It was shown in [3] (see also [4, 5, 6]) that the inclusion of transverse gauge links at light-cone infinity is indispensable for the restoration of full gauge invariance of the TMD PDFs in the light-cone gauge. Otherwise the gauge freedom in the light-cone gauge is not completely exhausted and, therefore, the light cone gauge is insufficient to trivialize the interaction of the struck quark with the gluon field of the spectators. Moreover, the transverse gauge link is responsible for the final (initial) state interactions.

2 Gauge-contour dependent TMD PDFs

We begin with the operator definition of the TMD distribution of a quark with momentum $k_\mu = (k^+, k^-, \mathbf{k}_\perp)$ in a quark with momentum $p_\mu = (p^+, p^-, \mathbf{0}_\perp)$, with decomposed gauge

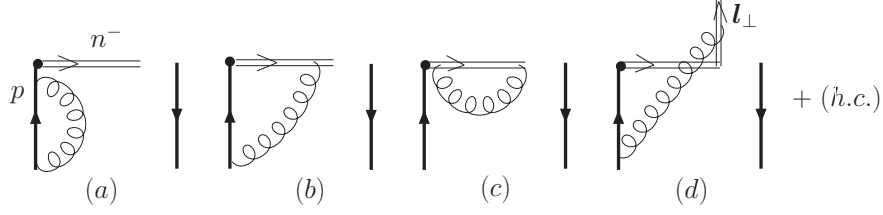


Figure 1: One-loop radiative corrections (curly lines) contributing UV-divergences to the TMD PDF in a general covariant gauge, with double lines denoting gauge links. Diagrams (b) and (c) are absent in the light-cone gauge, while the Hermitian conjugate (“mirror”) diagrams (not shown) are abbreviated by $(h.c.)$.

contours going through light-cone infinity:

$$\begin{aligned}
 f_{q/q}(x, \mathbf{k}_\perp) = & \frac{1}{2} \int \frac{d\xi^- d^2 \xi_\perp}{2\pi(2\pi)^2} \exp(-ik^+ \xi^- + i\mathbf{k}_\perp \cdot \xi_\perp) \langle q(p) | \bar{\psi}(\xi^-, \xi_\perp) [\xi^-, \xi_\perp; \infty^-, \xi_\perp]^\dagger \\
 & \times [\infty^-, \xi_\perp; \infty^-, \infty_\perp]^\dagger \gamma^+ [\infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp] [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \\
 & \times \psi(0^-, \mathbf{0}_\perp) | q(p) \rangle \big|_{\xi^+=0} .
 \end{aligned} \tag{1}$$

Here the gauge links, lightlike and transverse, respectively, are defined by the following path-ordered exponentials

$$\begin{aligned}
 [\infty^-, \mathbf{z}_\perp; z^-, \mathbf{z}_\perp] & \equiv \mathcal{P} \exp \left[ig \int_0^\infty d\tau \, n_\mu^- A_a^\mu t^a (z + n^- \tau) \right] \\
 [\infty^-, \infty_\perp; \infty^-, \xi_\perp] & \equiv \mathcal{P} \exp \left[ig \int_0^\infty d\tau \, \mathbf{l} \cdot \mathbf{A}_a t^a (\xi_\perp + \mathbf{l} \tau) \right] ,
 \end{aligned} \tag{2}$$

where the two-dimensional vector \mathbf{l} is arbitrary with no influence on the (local) anomalous dimensions we are interested in.

We have calculated [7] the one-gluon exchange contributions to the (unpolarized) TMD PDF of a quark in a quark and identified their UV divergences (see Fig. 1). It turns out that in the light-cone gauge, those contributions stemming from the interactions with the gluon field of the transverse gauge link cancel all terms that bear a dependence on the pole prescription applied to go around the light-cone singularities of the gluon propagator. As such, we have employed the retarded, advanced, and principal-value prescriptions (see for details [7]). Taking into account the Hermitian conjugate contributions, we

have identified an UV-divergent contribution that is absent in covariant gauges (e.g., the Feynman gauge) and can be conceived as originating from a non-trivial cusp-like junction point of the individual (transverse) gauge contours in the TMD PDF. Referring for details to [7], we here display only the complete UV-divergent part of the TMD PDF:

$$\begin{aligned}\Sigma_{\text{UV}}^{(a+d)}(p, \mu, \alpha_s; \epsilon) &= -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \left[\frac{1}{4} - \frac{\gamma^+ \hat{p}}{2p^+} \left(1 + \ln \frac{\eta}{p^+} - \frac{i\pi}{2} - i\pi C_\infty + i\pi C_\infty \right) \right] \\ &= -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \left[1 - \frac{\gamma^+ \hat{p}}{2p^+} \left(1 + \ln \frac{\eta}{p^+} - \frac{i\pi}{2} \right) \right].\end{aligned}\quad (3)$$

To complete the argument, take into account that $\frac{\gamma^+ \hat{p}}{2p^+} = \gamma^+$ and recall that we have to include the *mirror h.c.* counterparts of the evaluated diagrams. These yield complex-conjugated contributions, so that the imaginary terms mutually cancel. Hence, the UV-divergent part of diagrams (a) and (d) contains only contributions due to the p^+ -dependent term:

$$\Sigma_{\text{UV}}^{(a+d)}(\alpha_s, \epsilon) = 2 \frac{\alpha_s}{\pi} C_F \left[\frac{1}{\epsilon} \left(\frac{3}{4} + \ln \frac{\eta}{p^+} \right) - \gamma_E + \ln 4\pi \right]. \quad (4)$$

There is an extra anomalous dimension associated with the p^+ -dependent term which at the one-loop level reads $\left(\gamma = \frac{\mu}{2} \frac{1}{Z} \frac{\partial \alpha_s}{\partial \mu} \frac{\partial Z}{\partial \alpha_s} \right)$

$$\gamma_{1\text{-loop}}^{\text{LC}} = \frac{\alpha_s}{\pi} C_F \left(\frac{3}{4} + \ln \frac{\eta}{p^+} \right) = \gamma_{\text{smooth}} - \delta\gamma. \quad (5)$$

The renormalization effect due to the gluon corrections on the cusped junction point of the contours gives rise to the anomalous-dimension defect $\delta\gamma$ that has to be compensated in order to recover the same expression as in a covariant gauge according to the factorization proof. Moreover, it entails a modification in the multiplication rule for gauge links, according to [7]

$$\gamma\mathcal{C} = \gamma\mathcal{C}_1^\infty \cup \mathcal{C}_2^\infty + \gamma_{\text{cusp}} \iff [2, 1|\mathcal{C}] = [2, \infty|\mathcal{C}_2^\infty]^\dagger [\infty, 1|\mathcal{C}_1^\infty] e^{i\Phi_{\text{cusp}}}. \quad (6)$$

The crucial point here is to understand that the defect of the anomalous dimension can be identified with the universal cusp anomalous dimension [8], i.e.,

$$\begin{aligned}\gamma_{\text{cusp}}(\alpha_s, \chi) &= \frac{\alpha_s}{\pi} C_F (\chi \coth \chi - 1), \\ \frac{d}{d \ln p^+} \delta\gamma &= \lim_{\chi \rightarrow \infty} \frac{d}{d\chi} \gamma_{\text{cusp}}(\alpha_s, \chi) = \frac{\alpha_s}{\pi} C_F.\end{aligned}\quad (7)$$

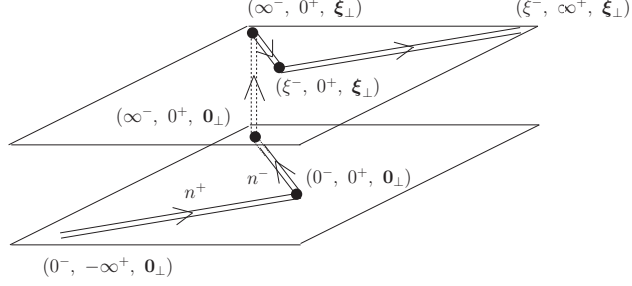


Figure 2: Integration contour associated with the additional soft counter term.

Then, to dispense with the anomalous-dimension artefact, the original TMD PDF will be redefined in terms of extra eikonal factors akin to the soft counter terms of Collins and Hautmann [9]:

$$R \equiv \Phi(p^+, n^- | 0) \Phi^\dagger(p^+, n^- | \xi) \quad (8)$$

with eikonal factors given by

$$\Phi(p^+, n^- | 0) = \left\langle 0 \left| \mathcal{P} \exp \left[ig \int_{\mathcal{C}_{\text{cusp}}} d\zeta^\mu t^a A_\mu^a(\zeta) \right] \right| 0 \right\rangle, \quad (9)$$

$$\Phi^\dagger(p^+, n^- | \xi) = \left\langle 0 \left| \mathcal{P} \exp \left[-ig \int_{\mathcal{C}_{\text{cusp}}} d\zeta^\mu t^a A_\mu^a(\xi + \zeta) \right] \right| 0 \right\rangle. \quad (10)$$

The soft counter term R has to be evaluated along the cusped integration contour (n_μ^- : minus light-cone vector)

$$\mathcal{C}_{\text{cusp}} : \zeta_\mu = \{ [p_\mu^+ s, -\infty < s < 0] \cup [n_\mu^- s', 0 < s' < \infty] \cup [\mathbf{l}_\perp \tau, 0 < \tau < \infty] \}, \quad (11)$$

depicted in Fig. 2. The main characteristic of this contour is the jump in the four-velocity $v_1 = p^+$ (parallel to the plus light-cone ray) at the origin to $v_2 = n^-$ (parallel to the minus light-cone ray), and creating an angle-dependence via $(v_1 \cdot v_2) = p^+$. Hence, the contour \mathcal{C} is cusped with an angle $\chi \sim \ln p^+ = \ln(p \cdot n^-)$. This way, we arrive at the

following redefined expression for the TMD PDF:

$$\begin{aligned}
f_{q/q}^{\text{mod}}(x, \mathbf{k}_\perp; \mu, \eta) &= \frac{1}{2} \int \frac{d\xi^- d^2 \boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \exp(-ik^+ \xi^- + i\mathbf{k}_\perp \cdot \boldsymbol{\xi}_\perp) \\
&\times \left\langle q(p) | \bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger \right. \\
&\times [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp] \\
&\times [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \psi(0^-, \mathbf{0}_\perp) | q(p) \rangle \\
&\times \left[\Phi(p^+, n^- | 0^-, \mathbf{0}_\perp) \Phi^\dagger(p^+, n^- | \xi^-, \boldsymbol{\xi}_\perp) \right], \tag{12}
\end{aligned}$$

which is the core result of our analysis.

3 Conclusions

We have given a modified definition of the TMD PDF for the unpolarized case which incorporates additional eikonal factors in order to cancel an anomalous-dimension contribution, stemming from the cusped-like junction of the gauge contours at light cone infinity in the transverse direction. These transverse gauge links are indispensable for the sake of full gauge invariance in the light-cone gauge, but would yield without these corrective eikonal factors to results that differ from those in covariant gauges in line with factorization. Integrating over the transverse momentum, our expression (12) provides an integrated PDF, which obeys the DGLAP evolution equation, without any dependence on pole prescriptions or the gauge-contour obstructions.

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